

Faddeev calculations of the $\bar{K}NN$ system with chirally-motivated $\bar{K}N$ interaction.

II. The K^-pp quasi-bound state.

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Abstract

New calculations of the quasi-bound state in the K^-pp system using Faddeev-type equations in AGS form with coupled $\bar{K}NN$ and $\pi\Sigma N$ channels were performed. A chiral $\bar{K}N$ potential together with phenomenological models of $\bar{K}N$ interaction with one- and two-pole structure of the $\Lambda(1405)$ resonance were used. All three potentials reproduce experimental data on low-energy K^-p scattering and kaonic hydrogen with the same level of accuracy. New method of calculating the subthreshold resonance position and width in a three-body system was proposed and used together with the direct search of the resonance pole. We obtained binding energy of the K^-pp quasi-bound state ~ 32 MeV with the chirally motivated and $47 - 54$ MeV with the phenomenological $\bar{K}N$ potentials. The width is about 50 MeV for the two-pole models of the interaction, while the one-pole potential gives ~ 65 MeV width. The question of using an energy dependent potential in few-body calculations is discussed in detail. It is shown that “self-consistent” variational calculations using such a potential are unable to produce a reasonable approximation to the exact result.

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I. INTRODUCTION

Exotic nuclei and atoms with non-zero strangeness attract attention from theorists and experimentalists. An assumption, that a quasi-bound state could exist in the lightest $\bar{K}NN$ system, appeared quite a long time ago [1], the interest to the topic was renewed after G-matrix calculations of several few-body antikaonic-nucleus systems [2]. Many theoretical calculations were performed after that, mostly focusing on the lightest K^-pp system. All of them used different few-body methods and two-body inputs, as a result the predicted binding energies and widths of the quasi-bound state differ substantially. The theoretical results agree only in the fact, that the quasi-bound state really can exist in the K^-pp system.

The first Faddeev calculations of a quasi-bound state in the K^-pp system with coupled channels were performed in [3, 4] and [5]. The authors of [5] then repeated their calculation in [6, 7] with energy dependent and independent versions of a chirally motivated $\bar{K}N$ potential. Other methods with less accurate treatment of the few-body dynamics were used for investigation of the K^-pp system after the Faddeev calculations, in particular, one-channel variational approaches [8–10] and some others.

The first experimental evidence of a quasi-bound state observation in K^-pp occurred in the FINUDA experiment [11] at the DAΦNE e^+e^- collider. New analyses of old experiments, such as OBELIX [12] at CERN and DISTO [13] at SATURNE also claim the observation of the state. The experimental results like the theoretical ones differ from each other, moreover, their binding energies and widths are far from all theoretical predictions. Since the question of the possible existence of the quasi-bound state in K^-pp system is still highly actual, new experiments are being planned and performed by HADES [14] and LEPS [15] Collaborations, in J-PARC E15 [16] and E27 [17] experiments.

The theoretical works differ not only in methods of treatment of the few-body system and models of two-body interactions, but also in accuracy of reproducing experimental data on K^-p scattering and kaonic hydrogen by the $\bar{K}N$ potentials. The $\bar{K}N$ interaction, which is the most important for the $\bar{K}NN$ system, is usually described either by pure phenomenological or by chirally motivated potentials. In particular, our previous calculations of the quasi-bound state in the $\bar{K}NN$ system [3, 4] were performed with an early phenomenological model of $\bar{K}N$ interaction. In a series of subsequent works devoted to the K^-d system [18–20] more accurate phenomenological models of $\bar{K}N$ interaction were constructed and used.

The potentials with coupled $\bar{K}N$ and $\pi\Sigma$ channels reproduce low-energy data on K^-p scattering and kaonic hydrogen and have a one-pole and a two-pole structure of the $\Lambda(1405)$ resonance, while the potential from [3, 4] has only one pole. In addition, a chirally motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential, which reproduces the experimental data with the same accuracy as the phenomenological ones, was constructed [21]. We used all three potentials in the new calculations of the K^-pp quasi-bound state and thereby investigated the dependence of the three-body results on the models of the $\bar{K}N$ interaction.

Faddeev equations in AGS form and the two-body input, which were used in the calculations, are shortly described in the next section together with the methods of evaluating the binding energy and width of a quasi-bound state. Two methods were used: the direct search of the pole position with contour rotation (described in Subsection II A) and a new method of finding a subthreshold resonance (II B). Our exact results are presented and discussed in Section III, which also contains our results of approximate calculations, performed additionally for comparison of our characteristics of the K^-pp quasi-bound states with those, obtained by other authors. Section IV is devoted to the question of using of an energy dependent potential in few-body equations. Series of additional calculations were performed to investigate the applicability of the “self-consistent” method of obtaining an “averaged $\bar{K}N$ energy in $\bar{K}NN$ system” used in the variational calculations. Our conclusions are drawn in the last section.

II. METHOD

The Faddeev equations in Alt-Grassberger-Sandhas form for the three-body system with coupled $\bar{K}NN$ and $\pi\Sigma N$ channels are described in detail in our previous papers [4, 19]. The equations are written for separable two-body potentials, they are properly antisymmetrized. The homogeneous system of 10 integral equations schematically can be written as

$$X_i(p) = \int_0^\infty Z_{ij}(p, p'; z) \tau_j \left(z - \frac{p'^2}{2\mu_j} \right) X_j(p') dp', \quad (1)$$

where X_i is an unknown function and τ_j is the energy dependent part of a two-body T -matrix describing the interaction of the particles (ik) (as is usual for Faddeev equations, $i \neq j \neq k$ is assumed), corresponding to a separable potential

$$V(k, k') = g(k) \lambda g(k') \quad \longrightarrow \quad T(k, k'; z^{(2)}) = g(k) \tau(z^{(2)}) g(k'). \quad (2)$$

The energy $z^{(2)}$ in Eq.(2) is an energy of a two-body system, while z in Eq.(1) is the three-body energy. Momentum k in Eq.(2) describes motion of a pair of particles, while p in Eq.(1) is a momentum of relative motion of a particle in respect to a pair. All additional indices and summations in the Eqs. (1,2) are omitted.

All our potentials are s -wave isospin dependent ones. We used three different models describing the $\bar{K}N$ interaction, which is the most important for the $\bar{K}NN$ system. Two phenomenological potentials with one-pole $V_{\bar{K}N-\pi\Sigma}^{1,SIDD}$ and two-pole $V_{\bar{K}N-\pi\Sigma}^{2,SIDD}$ structure of the $\Lambda(1405)$ resonance from [20] describe directly coupled $\bar{K}N$ and $\pi\Sigma$ channels, while the $\pi\Lambda$ channel is taken into account effectively through complex value of one of the strength parameters. Our chirally motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential with three coupled channels is described in I [21]. All three models are equally good in reproducing experimental data on low-energy K^-p scattering and kaonic hydrogen. In particular, they reproduce elastic and inelastic K^-p cross-sections and threshold branching ratio γ . The remaining threshold branching ratios R_c and R_n are reproduced by the $V_{\bar{K}N}^{Chiral}$ directly, while auxiliary $R_{\pi\Sigma}$ constructed from R_c and R_n is reproduced by the phenomenological potentials instead due to absence of the directly coupled $\pi\Lambda$ channel. All three models of $\bar{K}N$ interaction give values for $1s$ level shift and width of kaonic hydrogen, which are in agreement with the most recent experimental results from SIDDHARTA Collaboration [22].

The remaining two-body potentials, used the three-body calculation, are described in [19]. The two-term TSA-B NN potential reproduces phase shifts of Argonne V18 potential, therefore, is repulsive at short distances. It also gives proper NN scattering length, effective range and binding energy of deuteron. As for the model of ΣN interaction, we used the spin independent version of the exact optical potential corresponding to the model with coupled ΣN and ΛN channels. The two-channel $\Sigma N - \Lambda N$ potential reproduces low-energy experimental ΣN and ΛN cross-sections, the exact optical $\Sigma N(-\Lambda N)$ potential, which we used, has exactly the same elastic ΣN amplitude as the two-channel model.

The position z_0 of a quasi-bound state in the three-body problem is usually defined as $\lambda(z_0) = 1$, where $\lambda(z_0)$ is an eigenvalue of the kernel of Faddeev equations. In practice this amounts to solving the equation $\text{Det}(z_0) = 0$, where $\text{Det}(z)$ is the determinant of the linear system, obtained after discretization of the integral equations Eq. (1). We used two different types of discretization. One is based on the method of quadratures, another one uses a cubic spline expansion. All our results obtained with these two methods are equal, coinciding in

3 – 4 significant digits. We also used two methods of searching the complex pole position in a three-body system.

A. Direct pole search with contour rotation

Applying the coupled-channel Faddeev formalism for the search of K^-pp quasi-bound state at a complex energy one has to be aware of the specific requirements of finding a three-body resonance pole on the proper Riemann sheet. The problem of proper analytic continuation of the momentum space Faddeev equations from the physical energy sheet has an extended literature, see e.g. [23–27]. It has been established, that the correct analytic continuation to the closest unphysical energy sheet can be achieved by moving the momentum integration into the complex plane. Different integration contours were proposed in the literature. We chose the one suggested in [27], which is a ray in the fourth quadrant of the complex plane. Along the ray the momentum variable p' from Eq.(1) must satisfy the condition $|\text{Arg}(p')| > |\text{Arg}(z_0)|/2$. Deformation of the integration contour in this way ensures, that the complex solution of the equation will be found on the correct energy sheet.

B. $1/|\text{Det}|^2$ method

We found also another way to locate the K^-pp quasi-bound state, which avoids integration in the complex plane. Since the function $\text{Det}(z)$ has a zero at the quasi-bound state position z_0 , the function $1/\text{Det}(z)$, which enters all amplitudes through the inverse Faddeev matrix, can be written as

$$\frac{1}{\text{Det}(z)} = \frac{d(z)}{z - z_0}. \quad (3)$$

When energy z in Eq.(3) is taken on the real axes, the function $1/|\text{Det}(z)|^2$ has a Breit-Wigner form with $d(z)$ accounting for a background. Therefore, calculating $\text{Det}(z)$ for real energies z , for which the integration on momentum p' in Eq.(1) can be kept on the real axis, and fitting $1/|\text{Det}(z)|^2$ function to a Breit-Wigner curve we can get information on the resonance position and width. Obviously, this procedure works only if the resonance is well pronounced, i.e. isolated and not too wide. Fortunately, our K^-pp quasi-bound state is of this type, as can be seen on Fig.1, where we show the results of applying this method. The function $1/|\text{Det}(z)|^2$, calculated on the real energy axis using all three our $\bar{K}N$ potentials,

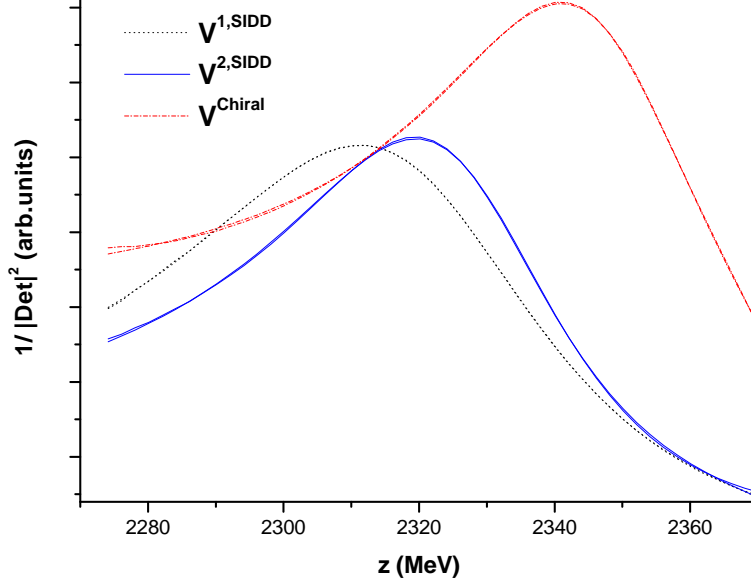


FIG. 1: Function $1/|\text{Det}(z)|^2$ calculated with the one-pole $V_{\bar{K}N-\pi\Sigma}^{1,SIDD}$ (black dotted line), two-pole $V_{\bar{K}N-\pi\Sigma}^{2,SIDD}$ phenomenological potentials (blue solid line) and chirally motivated $V_{\bar{K}N-\pi\Sigma-\pi\Lambda}^{\text{Chiral}}$ potential (red dash-dotted line). Breit-Wigner fits for all three functions are almost indistinguishable from the original lines.

is demonstrated. The Breit-Wigner fits to the curves are also plotted, however, they are almost indistinguishable from the original lines. The fits were done with a background function, which is quadratic in energy. It is remarkable, that the shape of the $1/|\text{Det}(z)|^2$ function on the real energy axis does not depend on the actual method of discretization of the integral equations. We obtained almost strictly coinciding Breit-Wigner parameters from quadratures and cubic spline expansion discretizations in spite of the fact, that the determinants themselves were strongly different.

Since complex root finding is a difficult task, the Breit-Wigner values can serve as a good starting point for it. On the other hand, it is a good test of the directly found pole position, which is free from the possible uncertainty of the proper choice of the Riemann sheet.

TABLE I: Pole positions z_{K^-pp} (in MeV, the real part is measured from the $\bar{K}NN$ threshold) of the quasi-bound state in the K^-pp system: the results of the direct pole search and of the Breit-Wigner fit of the $1/|\text{Det}(z)|^2$ function at real energy axis. The AGS calculations performed with the one-pole $V_{\bar{K}N-\pi\Sigma}^{1,SIDD}$, two-pole $V_{\bar{K}N-\pi\Sigma}^{2,SIDD}$ phenomenological potentials from [20] and the chirally-motivated $V_{\bar{K}N-\pi\Sigma-\pi\Lambda}^{\text{Chiral}}$ potential from [21] are demonstrated.

	Direct pole search	BW fit of $1/ \text{Det}(z) ^2$
with $V_{\bar{K}N}^{1,SIDD}$, [20]	$-53.3 - i\,32.4$	$-54.0 - i\,33.3$
with $V_{\bar{K}N}^{2,SIDD}$, [20]	$-47.4 - i\,24.9$	$-46.2 - i\,25.9$
with $V_{\bar{K}N}^{\text{Chiral}}$, [21]	$-32.2 - i\,24.3$	$-30.3 - i\,23.3$

III. RESULTS

Pole positions of the K^-pp quasi-bound state, obtained from the direct search in the complex plane and from the Breit-Wigner fit with three our $\bar{K}N$ potentials are shown in Table I. The one- $V_{\bar{K}N}^{1,SIDD}$ and two-pole $V_{\bar{K}N}^{2,SIDD}$ phenomenological interaction models from [20] together with the chirally motivated potential $V_{\bar{K}N}^{\text{Chiral}}$ from [21] were used. It is seen, that the results obtained using the two methods of pole position search are quite close, indicating, that the methods supplement each other.

The most striking feature of the results, shown in the table, is the large difference between the binding energies of the quasi-bound states obtained from the phenomenological, especially the one-pole version, and the chirally motivated $\bar{K}N$ potentials. This probably is due to the energy dependence of chirally motivated model of the interaction. The available experimental data, to which all three potentials were fitted with approximately equal accuracy are close to the $\bar{K}N$ threshold. While the phenomenological models of the $\bar{K}N$ interaction are unchanged, when the K^-pp quasi-bound state is calculated, the energy dependence of the chirally motivated potential reduces the attraction for the lower energies in the $\bar{K}NN$ quasi-bound state region, thus producing states with less binding.

It is not quite clear, why the widths of the two-pole models of $\bar{K}N$ interaction are almost coinciding, while the one-pole $V_{\bar{K}N}^{1,SIDD}$ potential gives much larger width. The difference might be connected with the different pole structure of the corresponding $\bar{K}N$ interaction models: while the highest poles of the two-pole $V_{\bar{K}N}^{2,SIDD}$ and chirally motivated $V_{\bar{K}N}^{\text{Chiral}}$

TABLE II: Binding energy B_{K^-pp} (MeV) and width Γ_{K^-pp} (MeV) of the quasi-bound state in the K^-pp system. The results obtained from AGS calculation with the one-pole $V_{\bar{K}N-\pi\Sigma}^{1,SIDD}$, two-pole $V_{\bar{K}N-\pi\Sigma}^{2,SIDD}$ phenomenological potentials from [20] and the chirally-motivated $V_{\bar{K}N-\pi\Sigma-\pi\Lambda}^{Chiral}$ potential from [21] are demonstrated. Other theoretical results of Faddeev [4, 7] and variational [9, 10] calculations are also shown.

	B_{K^-pp}	Γ_{K^-pp}
<u>Present AGS:</u>		
with $V_{\bar{K}N}^{1,SIDD}$, [20]	53.3	64.8
with $V_{\bar{K}N}^{2,SIDD}$, [20]	47.4	49.8
with $V_{\bar{K}N}^{Chiral}$, [21]	32.2	48.6
<u>Previous AGS:</u>		
SGMR [4]	55.1	100.2
IKS [7] with $V_{\bar{K}N}^{E-indep}$	44 - 58	34 - 40
IKS [7] with $V_{\bar{K}N}^{E-dep}$	9 - 16	34 - 46
	67 - 89	244 - 320
<u>Variational:</u>		
DHW [9]	17 - 23	40 - 70
BGL [10]	15.7	41.2

potentials lie close to each other, the pole position of the one-pole phenomenological model is much closer to the $\bar{K}N$ threshold (see Table 2 of [20] and Eq.(12) of [21]).

Such a large difference between the “phenomenological” and “chiral” results is opposite to the results of I [21], where low-energy K^-d scattering and 1s level shift and width of kaonic deuterium were calculated using the same equations (surely, inhomogeneous ones with correspondingly changed quantum numbers) and input. In that case the three-body observables obtained with the three $\bar{K}N$ potentials turned out to be very close each to other. It can be due to the fact, that those three-body values were calculated near the $\bar{K}NN$ threshold while the K^-pp pole positions are far below it.

Our three binding energy B_{K^-pp} and width Γ_{K^-pp} values of the K^-pp quasi-bound state are compared in Table II with other theoretical results. In particular, the results obtained in

our previous Faddeev calculation [4], the most recent results of alternative calculation using the same equations [7] with several chirally motivated $\bar{K}N$ potentials are shown together with two variational results [9, 10]. The new result with one-pole $V_{\bar{K}N}^{1,SIDD}$ potential has binding energy, which is very close to our previous one [4], which used the same model of $\bar{K}N$ interaction. The difference in widths could be explained by low accuracy of the older $\bar{K}N$ potential.

Coupled-channel AGS equations were solved in [7] with chirally motivated energy dependent and independent $\bar{K}N$ potentials. Therefore, in principle, their calculation with the energy dependent version of the $\bar{K}N$ potential $V_{\bar{K}N}^{E-dep}$ should give a result, which is close to ours with chirally motivated model of interaction $V_{\bar{K}N}^{Chiral}$. It is seen, however, that only their width is comparable to our Γ_{K^-pp} , while the binding energy obtained in [7] is much smaller than ours. The reason of the difference is, probably, an approximation used in the chirally motivated models used in [6, 7]. Namely, the energy-dependent square root factors, responsible for the correct normalization of the amplitudes, are replaced by constant masses. This can be reasonable for the highest $\bar{K}N$ channel, however, it is certainly a poor approximation for the lower lying $\pi\Sigma$ and $\pi\Lambda$ channels. We checked the role of this approximation in the AGS calculations, the obtained pole position, corresponding to the quasi-bound state,

$$z_{K^-pp}^{Const.norm} = -25.0 - i 23.4 \text{ MeV} \quad (4)$$

really has much smaller binding energy than the original one, see Table II. The remaining difference between the results could be explained by the higher accuracy of reproducing experimental K^-p data by our chirally motivated $\bar{K}N$ potential.

We did not find the second pole in the K^-pp system reported in [7] in the corresponding region for either of the three $\bar{K}N$ potentials.

There are a few problematic points in [9, 10], too. First of all, the variational calculation was performed solely in the $\bar{K}NN$ sector, therefore the absorption into the $\pi\Sigma N$ and $\pi\Lambda N$ channels should be taken into account through the imaginary part of an optical or complex potential. We checked an accuracy of use of the exact optical $\bar{K}N$ potential, which gives exactly the same elastic $\bar{K}N$ amplitude as the original potential with coupled channels, and performed one-channel AGS calculations for three our $\bar{K}N$ potentials. The “exact optical” pole positions

$$z_{K^-pp}^{1,SIDD,Opt} = -54.2 - i 30.5 \text{ MeV} \quad (5)$$

$$z_{K^-pp}^{2,SIDD,Opt} = -47.4 - i 23.0 \text{ MeV} \quad (6)$$

$$z_{K^-pp}^{Chiral,Opt} = -32.9 - i 24.4 \text{ MeV} \quad (7)$$

differ only slightly from the full coupled-channel results from Table I. Therefore, the one-channel Faddeev calculation with exact optical potential could be quite satisfactory approximation to the full calculation with coupled channels. The authors of variational calculations [9, 10] used a one-channel $\bar{K}N$ potential, derived from a chirally motivated model of interaction with many coupled channels. However, the potential cannot be called “the exact optical” since Gauss form-factors were additionally introduced into the potential. It is not quite clear, how this one-channel potential is connected to the original one and whether it still reproduces some experimental $\bar{K}N$ data.

Moreover, the position of the quasi-bound state was determined in [9, 10] only from the real part of this $\bar{K}N$ potential, as a real bound state, while the width was estimated as the expectation value of the imaginary part of the potential. This, essentially perturbative treatment of the inelasticity might be justified for quite narrow resonances, but the quasi-bound state under consideration is certainly not of this type.

Another serious problem of the variational calculations is their method of treatment of the energy dependence of the $\bar{K}N$ potential in the few-body calculations. Our results already show that the energy dependence of the chirally motivated model of $\bar{K}N$ interaction has a crucial effect on the K^-pp quasi-bound state position. Therefore, the question deserves special attention.

IV. ENERGY DEPENDENT $\bar{K}N$ POTENTIAL IN FEW-BODY CALCULATIONS

The basic problem is, that two-body energy is not a well defined quantity in more-than-two-body systems. Therefore some special effort is needed if energy dependent interactions are used in few-body calculations. Fortunately, momentum space Faddeev equations provide a framework, offering an exact treatment of this problem. It is seen from Eqs.(1) and (2) that the argument of the energy dependent part τ_j of the two-body T -matrix

$$z_j^{(2)} \equiv z - \frac{p'^2}{2\mu_j} \quad (8)$$

comes from embedding the two-body T -operator in three-body space. The kinetic energy of the third, non-interacting particle with momentum p' in Eq.(8) is extracted from the

three-body energy z . This is the so called spectator mechanism, through which two-body dynamics enters the three-body problem in Faddeev approach. The representation of a few-body system as an auxiliary subsystem of two interacting particles and a third, non-interacting spectator allows an exact definition of the two-particle energy at which the two-body T -matrix has to be evaluated. It is seen from Eq.(1), that since the spectator momentum p' is an integration variable, a range of two-particle energies $z_j^{(2)} \subset [z, -\infty)$ enters the three-body calculation.

Our results, shown in the previous section, were obtained in this way, allowing for a “dynamical” dependence on two-body energies. However, we made an exception: since the coupling constants depending on $z_j^{(2)}$ obviously become non-physical for $p' \rightarrow \infty$, we “froze” the energy dependence at the $\pi\Lambda$ threshold, where $\text{Re } z_j^{(2)} = m_\pi + m_\Lambda$, and kept these values, when p' was further increased under the integral. We think, that this procedure is non-contradictory to the spirit of chiral interactions, whose energy dependence is probably meant in a certain region near the channel thresholds, and definitely not below the lowest open channel. We checked, that this freezing does not change the pole position, situated far above the $\pi\Lambda$ threshold.

When chiral interactions are used in non-Faddeev few-body calculations, their energy dependence has to be accounted for. To treat this problem in coordinate space variational calculations, which can be performed only with fixed-energy two-body interactions, a method was invented in [8] and then used in [9, 10]. It is based on the assumption, that a definite two-particle energy $z_{\bar{K}N}$ exists, for which the chiral interaction fixed at this value produces the same K^-pp quasi-bound state, as the energy dependent one. A “self-consistent” iterative procedure was suggested, according to which the expectation value of a certain operator, calculated with the trial wave function and called “average $\bar{K}N$ energy in $\bar{K}NN$ system”, should give $z_{\bar{K}N}$. Apart from the fact that the quantity, with respect to which self-consistency is sought, is not free of some amount of arbitrariness, no single hint concerning the applicability or accuracy of this method is given.

We decided to investigate the effect of fixing the energy of the $\bar{K}N$ coupling functions on the $\bar{K}NN$ quasi-bound state position. Our results are shown in Fig.2, where the K^-pp quasi bound state pole trajectories, calculated with a fixed $z_{\bar{K}N}$ in the couplings of our chirally motivated $\bar{K}N$ potential are plotted. The curves correspond to changing $\text{Re } z_{\bar{K}N}$ up to -100 MeV, keeping $\text{Im } z_{\bar{K}N}$ fixed. $|\text{Re } z_{\bar{K}N}|$ values are marked on the plot near the corresponding

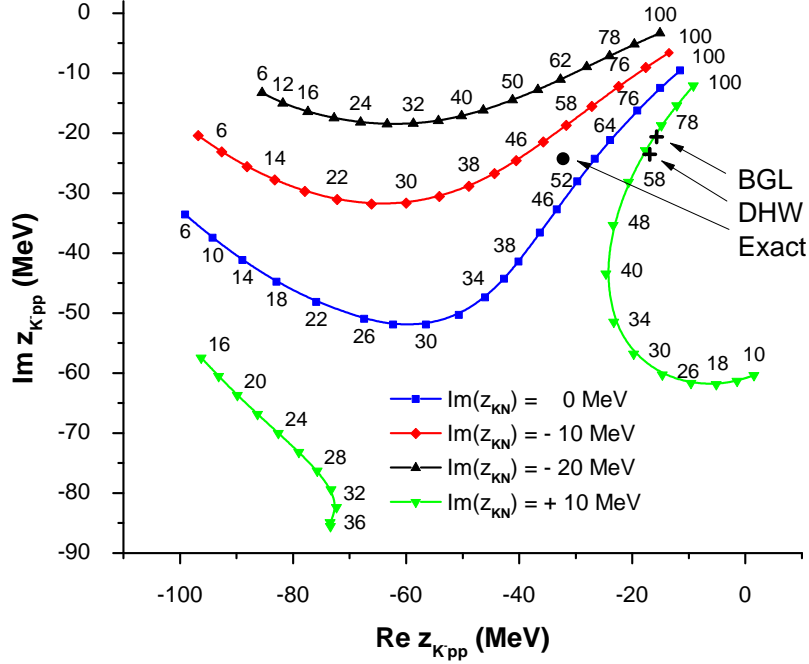


FIG. 2: Series of the quasi-bound system pole positions in the K^-pp system calculated with fixed energy of the chirally motivated $\bar{K}N$ potential. Each line contains results obtained with $\text{Im } z_{\bar{K}N} = -20$ MeV (black triangles up), $\text{Im } z_{\bar{K}N} = -10$ MeV (red diamonds), $\text{Im } z_{\bar{K}N} = 0$ MeV (blue squares) and $\text{Im } z_{\bar{K}N} = +10$ MeV (green triangles down) with $|\text{Re } z_{\bar{K}N}|$, changing up to 100 MeV (numbers near the lines). Exact result of Faddeev calculation with coupled channels (black circle) and two results of variational calculations (crosses) are also shown: BGL [10] and type I result with HNJH $\bar{K}N$ potential DHW [9].

points. The curve with $\text{Im } z_{\bar{K}N} = 0$ corresponds to the line, on which, according to [9, 10], a self-consistent procedure can find the correct quasi-bound state position. We see, that this claim is unjustified, especially for the self-consistent values of $\text{Re } z_{\bar{K}N}$ found in [9, 10]:

$$\text{Re } z_{\bar{K}N}^{DHW} = -39 \text{ MeV} \quad (9)$$

$$\text{Re } z_{\bar{K}N}^{BGL} = -43 \text{ MeV}. \quad (10)$$

The curves with non-zero $\text{Im } z_{\bar{K}N}$ show an interesting “inverse” behavior: with increasing $|\text{Im } z_{\bar{K}N}|$ the quasi-bound state becomes narrower. This is due to the fact, that $z_{\bar{K}N}$ enters

the diagonal couplings of the chiral potential with negative sign. Thus increasing of the negative $\text{Im } z_{\bar{K}N}$ by absolute value corresponds not to increasing of the absorption due to the open $\pi\Sigma N$ channel, but to its reduction.

From Fig.2 one can also conclude, that the imaginary part of $z_{\bar{K}N}$ influences not only the width of the quasi-bound state, but its position, too. Otherwise, the points on the curves, corresponding to equal $\text{Re } z_{\bar{K}N}$ values, should lie strictly below each other. The deviations from this pattern are apparent, especially towards the ends of trajectories.

If one asks the question, whether a similar curve can be found, which contains the exact pole, the answer is “yes”, with $\text{Im } z_{\bar{K}N} \simeq -7$ MeV, while the pole is found at $\text{Re } z_{\bar{K}N} \simeq -58$ MeV. This value of $z_{\bar{K}N}$ is rather far from the “self-consistent” ones, Eqs.(9,10). Moreover, it is also possible to find $z_{\bar{K}N}$ values, which yield the quasi-bound state positions, found in [9, 10], also shown on Fig.2. These values, however, have positive imaginary parts, which is hard to interpret. It is interesting to note that for positive fixed $\text{Im } z_{\bar{K}N}$ above a certain (critical) value the trajectories have two branches, as it is seen on the example with $\text{Im } z_{\bar{K}N} = +10$ MeV. As a consequence, some values of $z_{\bar{K}N}$ allow for two poles in the considered region of the energy plane.

In general, it looks like for any quasi-bound state location one can find a complex $z_{\bar{K}N}$, which in a coupled-channels Faddeev calculation leads to this pole position. However, even if we know, that there exists a $z_{\bar{K}N}$ giving the correct quasi-bound state pole, it is absolutely not clear, whether an operator can be defined, whose expectation value would give this $z_{\bar{K}N}$, at least approximately. Without such an operator no self-consistent scheme can be constructed to treat the energy dependence of the interaction.

V. CONCLUSIONS

We calculated $\bar{K}NN$ quasi-bound state positions for the two phenomenological and the chirally motivated models of the $\bar{K}N$ interaction, which all describe the available experimental K^-p data equally well. We found, that the quasi-bound states resulting from the phenomenological potentials lie about 15 – 20 MeV deeper, than those of the chirally motivated one. In our opinion, this is due to the energy dependence of the chiral interaction, leading to less attraction for lower energies. We obtained binding energy ~ 32 MeV for the chirally motivated and 47 – 54 MeV for the phenomenological $\bar{K}N$ potentials. The width is

about 50 MeV was obtained with the two-pole models of the interaction, while the one-pole potential gives ~ 65 MeV.

We proposed a new $1/|\text{Det}(z)|^2$ method of finding mass and width of a subthreshold resonance and demonstrated its efficiency.

We discussed in some detail, how energy dependence of the two-body interaction can be accounted for in few-body calculations. It was shown, that momentum space Faddeev integral equations allow an exact treatment of this energy dependence. On the contrary, coordinate space variational methods can use only energy independent interactions, therefore we performed a series of calculations with differently fixed two-particle $\bar{K}N$ energies $z_{\bar{K}N}$ in the couplings of the chirally motivated interaction. Our conclusion is, that the method used in [9, 10] is unable to define an “averaged” $z_{\bar{K}N}$, for which the fixed-energy chirally motivated interaction, even in a correct three-body calculation, can yield a K^-pp quasi-bound state position with any relation to the exact one.

First, our calculations show, that a real $z_{\bar{K}N}$ has absolutely no chance to reproduce or reasonably approximate the exact quasi-bound state position, even with correct treatment of the imaginary part of the interaction, unlike in [9, 10]. Second, the way, how the “self-consistent” value of (generally complex) $z_{\bar{K}N}$ is defined in the papers does not seem to guarantee, that the correct value will be reached or at least approximated. In view of the above considerations, the results of [9, 10] can be considered as rough estimates of what a really energy dependent $\bar{K}N$ interaction will produce in the K^-pp system. Similarly, the four-body results of [10] would hardly survive a comparison with an exact four-body calculation, which, however, still has to be done.

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